NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BHAM	**LEVEL:** 8
COURSE CODE: ANA801S	**COURSE NAME:** APPLIED NUMERICAL ANALYSIS
SESSION: JUNE 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 120 (to be converted to 100%)

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF S. A. REJU
MODERATOR:	PROF S. MOTSA

INSTRUCTIONS

1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencils.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [30 MARKS]

(a) Discuss the general Iterated quadrature rule for obtaining the following integral

$$I = \int_a^b f(x)dx \qquad (1.1)$$

and hence state the Iterated Trapezoidal Rule for (1.1). **[5]**

(b) From your Composite Trapezoidal rule in (a), state the Romberg's Method for solving (1.1) and hence using the unit interval [0, 1] for the integral

$$T(n) = \int_a^b f(x)dx$$

and step size

$$h = \frac{(b-a)}{n}$$

obtain the term for the recursive expression $T(2^n) = T(8)$ and the expression for $R(n, 0)$ denoting the Trapezoidal estimate with 2^n. **[16]**

(c) By just stating the Richardson's Extrapolation $R(n, m)$ employed in the Romberg's Table, show that

$$R(1,0) = \frac{1}{2}R(0,0) + \frac{1}{2}(b-a)f\left(\frac{a+b}{2}\right)$$

[9]

QUESTION 2 [30 MARKS]

(a) Discuss and derive the recursive scheme for the Forward Euler's Method, using any appropriate diagram for substantiating your discussion. **[13]**

(b) Consider the following IVP:

$$\frac{dy(t)}{dt} + 2y(t) = 3e^{-4t}, \quad y(0) = 1$$

Using a step size of h = 0.1 and $t_o = 0$, employ the method discussed in (1.1) to approximate up to the 5^{th} step, giving your solution in a table showing both the exact and the approximate solution at each step. **[17]**

QUESTION 3 [30 MARKS]

(a) Discuss the contrast between a quadrature rule and the adaptive rule. **[3]**

(b) Consider the integral **[27]**

$$\int_a^b f(x)dx = \int_1^3 e^{2x} sin(3x)dx$$

Using the Adaptive Simpson's Method and an error $\epsilon = 0.2$, obtain the approximate value of the above integral (for computational ease, using where appropriate the following as done in class):

$$\frac{1}{10}\left|S(a,b) - S(a,\frac{a+b}{2}) - S(\frac{a+b}{2},b)\right|$$

where

$$\int_a^b f(x)dx = (S(a,b) - \frac{h^5}{90}f^{(4)}(\xi), \ \ \xi\epsilon(a,b)$$

QUESTION 4 [30 MARKS]

(a) State the pseudo code for the Conjugate Gradient Method (CGM) for solving the nxn system of linear equations:

$$Ax = b$$

where A is a symmetric and positive definite matrix. **[10]**

(b) Consider the following system of linear equations:

$$\begin{bmatrix} 5 & -2 & 0 \\ -2 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ -10 \end{bmatrix}$$

Solve the above system using the Conjugate Gradient Method using the initial vector:

$$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[20]

END OF QUESTION PAPER TOTAL MARKS = 120